Name ANSWERS	Class

MATHS TEACHER HUB

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Parallel and perpendicular lines

(9 – 1) Topic booklet

Higher

These questions have been collated from previous years GCSE Mathematics papers.

You must have: Ruler graduated in centimetres and millimetres, protractor, pair of compasses, pen, HB pencil, eraser.	Total Marks

Instructions

- Use **black** ink or ball-point pen.
- **Fill in the boxes** at the top of this page with your name, centre number and candidate number.
- Answer **all** questions.
- Answer the questions in the spaces provided
  – *there may be more space than you need.*
- Diagrams are NOT accurately drawn, unless otherwise indicated.
- You must **show all your working out.**
- If the question is a **1H** question you are not allowed to use a calculator.
- If the question is a **2H** or a **3H** question, you may use a calculator to help you answer.

Information

- The marks for **each** question are shown in brackets
  – *use this as a guide as to how much time to spend on each question.*

Advice

- Read each question carefully before you start to answer it.
- Keep an eye on the time.
- Try to answer every question.
- Check your answers if you have time at the end.

Answer ALL questions
Write your answers in the space provided.
You must write down all the stages in your working.

6 The equation of the line L_1 is $y = 3x - 2$
The equation of the line L_2 is $3y - 9x + 5 = 0$

Show that these two lines are parallel.

L_2 $3y - 9x + 5 = 0$

$3y = 9x - 5$

$y = 3x - \frac{5}{3}$

Both L_1 and L_2 have a gradient of 3
so they are parallel.

(Total for Question 6 is 2 marks)

9 Here are the equations of two straight lines.

$$y = \frac{1}{2}x - 6 \qquad 6y = 3x + 7$$

Oscar says that these lines are parallel.

Is Oscar correct?
You must give a reason for your answer.

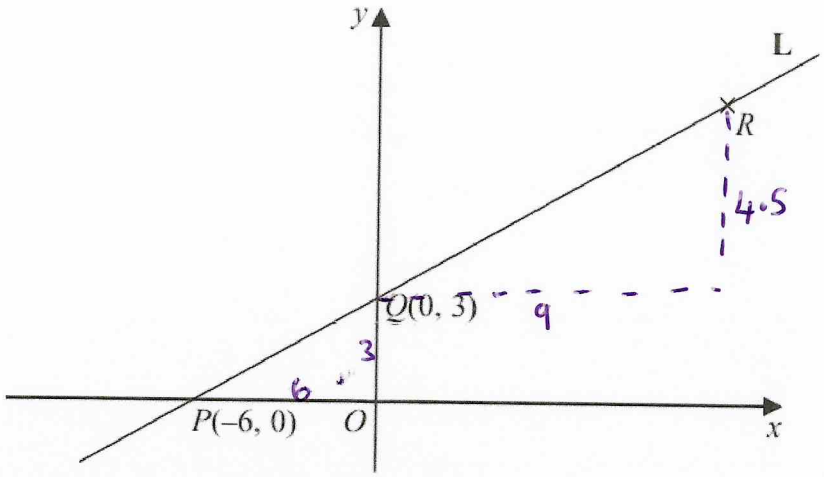
$y = \frac{3x}{6} + \frac{7}{6}$

$y = \frac{1}{2}x + \frac{7}{6}$

Oscar is correct, both lines have a gradient of $\frac{1}{2}$, which means they are parallel.

(Total for Question 9 is 2 marks)

11 Here is a sketch of the line **L**.



The points $P(-6, 0)$ and $Q(0, 3)$ are points on the line **L**.

The point R is such that PQR is a straight line and $PQ : QR = 2 : 3$

(a) Find the coordinates of R.

(9 , 7.5)

(2)

(b) Find an equation of the line that is perpendicular to **L** and passes through Q.

$L = y = \frac{1}{2}x + 3$

$Q = (0, 3)$

$L_2 = y = -2x + c$

$3 = -2(0) + c$

$3 = c$

$y = -2x + 3$

(3)

(Total for Question 11 is 5 marks)

12 The straight line **L** has equation $2y = 3x - 7$

Find an equation of the straight line perpendicular to **L** that passes through $(6, -5)$

x y

$L = y = \frac{3}{2}x - \frac{7}{2}$

$L_1 = y = -\frac{2}{3}x + C$

$-5 = -\frac{2}{3}(6) + C$

$-5 = -\frac{12}{3} + C$

$-5 = -4 + C$

$-1 = C$

$y = -\frac{2}{3}x - 1$

November 2023 – Paper 1H

(Total for Question 12 is 3 marks)

12 The equation of the line L_1 is $y = 2x + 3$
The equation of the line L_2 is $5y - 10x + 4 = 0$

Show that these two lines are parallel.

$L_2 = 5y - 10x + 4 = 0$

$5y = 10x - 4$

$y = \frac{10x}{5} - \frac{4}{5}$

$y = 2x - \frac{4}{5}$

Both L_1 and L_2 have a gradient of $2x$ so they are parallel.

June 2022 – Paper 2H

(Total for Question 12 is 2 marks)

15 The straight line L_1 has equation $y = 3x - 4$
The straight line L_2 is perpendicular to L_1 and passes through the point (9, 5)

Find an equation of line L_2

x y

$L_2 = y = -\frac{1}{3}x + C$

$5 = -\frac{1}{3}(9) + C$

$5 = -\frac{9}{3} + C$

$5 = -3 + C$

$8 = C$

$y = -\frac{1}{3}x + 8$

(Total for Question 15 is 3 marks)

15 The equation of line L_1 is $y = 2x - 5$
The equation of line L_2 is $6y + kx - 12 = 0$

L_1 is perpendicular to L_2

Find the value of k.
You must show all your working.

Gradient of $L_1 = 2$

Gradient of $L_2 = -\frac{1}{2}$

$6y + kx - 12 = 0$

$6y = -kx + 12$

$y = -\frac{kx}{6} + \frac{12}{6}$

$\downarrow$

$-\frac{1}{2}$

$k =$ 3

(Total for Question 15 is 3 marks)

16 The straight line **L** has the equation $3y = 4x + 7$

The point A has coordinates $(3, -5)$
x y

Find an equation of the straight line that is perpendicular to **L** and passes through A.

L_1 $3y = 4x + 7$

$y = \frac{4x}{3} + \frac{7}{3}$

L_2 $y = -\frac{3}{4}x + C$

$-5 = -\frac{3}{4}(3) + C$

$-5 = \frac{-9}{4} + C$

$-5 = -2\frac{1}{4} + C$

$-2\frac{3}{4} = C$

$y = -\frac{3}{4}x - 2\frac{3}{4}$

(Total for Question 16 is 3 marks)

x y

19 The point P has coordinates $(3, 4)$
The point Q has coordinates (a, b)

A line perpendicular to PQ is given by the equation $3x + 2y = 7$

Find an expression for b in terms of a.

$2y = -3x + 7$

$y = -\frac{3x}{2} + \frac{7}{2}$

PQ $y = \frac{2}{3}x + c$

$4 = \frac{2}{3}(3) + c$

$4 = \frac{6}{3} + c$

$4 = 2 + c$

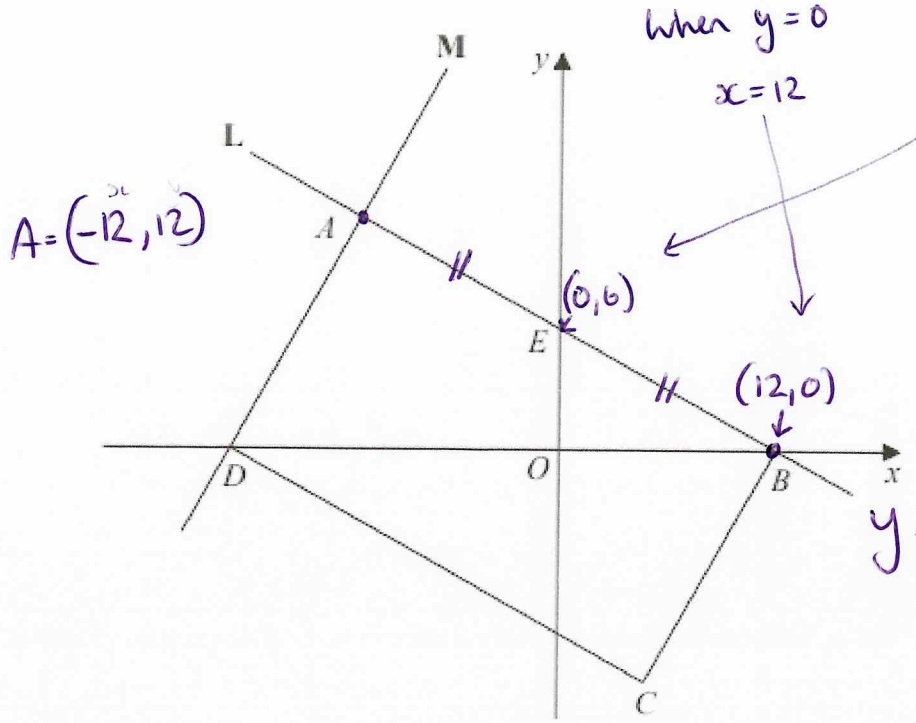
$2 = 2$

$y = \frac{2}{3}x + 2$

$b = \frac{2}{3}a + 2$

(Total for Question 19 is 5 marks)

19



when $y = 0$: $x = 12$

when $x = 0$: $2y = 12$, $y = 6$

$A = (-12, 12)$

$(0, 6)$

$(12, 0)$

$y = -\frac{1}{2}x + 6$

$ABCD$ is a rectangle.

A, E and B are points on the straight line **L** with equation $x + 2y = 12$
A and D are points on the straight line **M**.

$AE = EB$

Find an equation for **M**.

$2y = -x + 12$

$y = -\frac{1}{2}x + 6$

$m = -\frac{1}{2}x$

M = $y = 2x + c$

$12 = 2(-12) + c$

$12 = -24 + c$

$36 = c$

$y = 2x + 36$

(Total for Question 19 is 4 marks)

19 A triangle has vertices P, Q and R.

The coordinates of P are $(-3, -6)$
The coordinates of Q are $(1, 4)$
The coordinates of R are $(5, -2)$

M is the midpoint of PQ.
N is the midpoint of QR.

Prove that MN is parallel to PR.
You must show each stage of your working.

$M = (-3, -6) + (1, 4) = (-2, -2)$

$\div 2 \; (-1, -1)$

$N = (1, 4) + (5, -2) = (6, 2)$

$\div 2 \; (3, 1)$

$PR = (-3, -6) \overset{P}{} \longrightarrow (5, -2) \overset{R}{}$

$\frac{\Delta y}{\Delta x} = \frac{4}{8} = \frac{1}{2}$

$MN = (-1, -1) \overset{M}{} \longrightarrow (3, 1) \overset{N}{}$

$\frac{\Delta y}{\Delta x} = \frac{2}{4} = \frac{1}{2}$

PR gradient $= \frac{1}{2}$

MN gradient $= \frac{1}{2}$

Both lines have the same gradient so they are parallel.

(Total for Question 19 is 4 marks)

22 Given that the vector $a\begin{pmatrix}2\\6\end{pmatrix}+b\begin{pmatrix}8\\2\end{pmatrix}$ is parallel to the vector $\begin{pmatrix}13\\6\end{pmatrix}$

find an expression for b in terms of a.

$2a + 8b = 13x$ $\quad$ $6a + 2b = 6x$

$\frac{2}{13}a + \frac{8}{13}b = x$ $\quad$ $\frac{6a}{6} + \frac{2b}{6} = x$

$-\frac{2}{13}a$ $\quad$ $\frac{2}{13}a + \frac{8}{13}b = a + \frac{1}{3}b$ $\quad$ $-\frac{2}{13}a$

$-\frac{1}{3}b$ $\quad$ $\frac{8}{13}b = \frac{11}{13}a + \frac{1}{3}b$ $\quad$ $-\frac{1}{3}b$

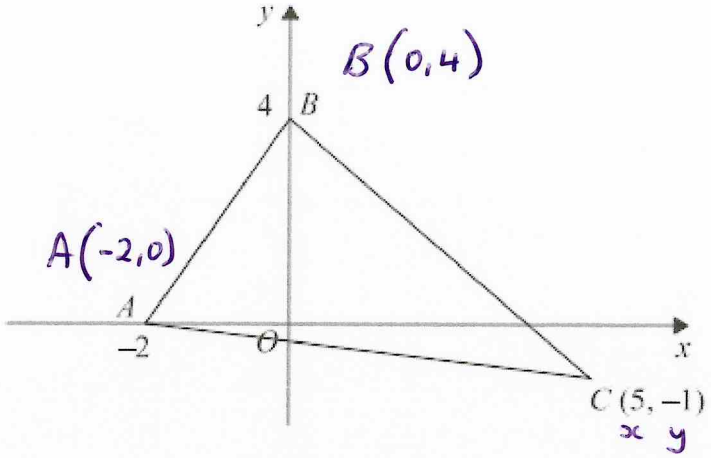
$\frac{11}{39}b = \frac{11}{13}a$

$b = 3a$

$b = 3a$

(Total for Question 22 is 3 marks)

23



Find an equation of the line that passes through C and is perpendicular to AB.

AB gradient

$\frac{\Delta y}{\Delta x} = \frac{4}{2} = 2$

Perpendicular to AB

gradient $= -\frac{1}{2}$

$y = -\frac{1}{2}x + C$

$-1 = -\frac{1}{2}(5) + C$

$-1 = -\frac{5}{2} + C$

$1.5 = C$

$y = -\frac{1}{2}x + 1.5$

(Total for Question 23 is 4 marks)

25 The straight line **L** has equation $3x + 2y = 17$

The point A has coordinates $(0, 2)$ (x y)
The straight line **M** is perpendicular to **L** and passes through A.

Line **L** crosses the y-axis at the point B.
Lines **L** and **M** intersect at the point C.

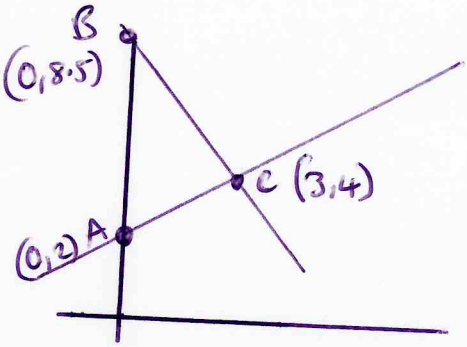
Work out the area of triangle ABC.
You must show all your working.

Point B (0, 8.5)

L $3x + 2y = 17$

$2y = -3x + 17$

$\boxed{y = -\frac{3}{2}x + \frac{17}{2}}$



M = $y = \frac{2}{3}x + c$

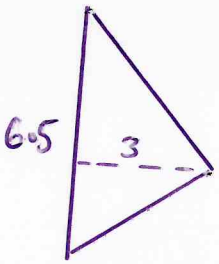
$2 = \frac{2}{3}(0) + c$

$2 = c$

$\boxed{y = \frac{2}{3}x + 2}$

Area

L and M intersect



$\frac{6.5 \times 3}{2} = 9.75$

$-\frac{3}{2}x + \frac{17}{2} = \frac{2}{3}x + 2$ (×2)

$-3x + 17 = \frac{4}{3}x + 4$ (×3)

$-9x + 51 = 4x + 12$ (−12)

$-9x + 39 = 4x$ (+9x)

$39 = 13x$

$3 = x$

Point C = (3, 4)

$y = \frac{2}{3}(3) + 2$

$y = 4$

9.75

(Total for Question 25 is 5 marks)